Module : 3

Phasors

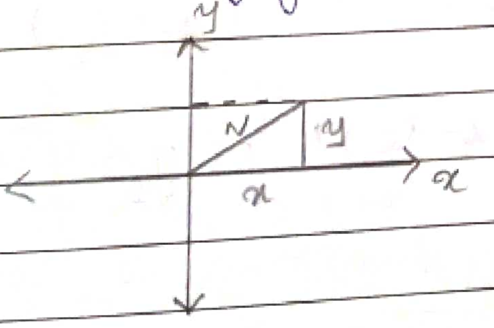
4 types of phasors are:

1) Rectangular form
2) Trigonometric form
3) Exponential form
4) Polar form

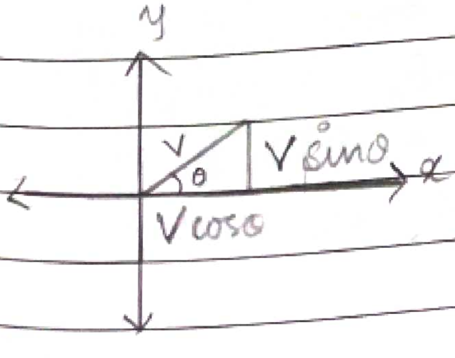
$j = \sqrt{-1}$, J is an operator it is used to express the 90° operation in counter clo-ckwise direction.

Rectangular Form

$v = x + jy$ $\{ j = i = \sqrt{-1} \}$



Trigonometric form



$v = V(\cos\theta + j\sin\theta) =$

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- **Exponential Form ($e^{j\theta}$)**

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$v = V e^{-j\theta}$$

- **Polar Form**

$$v = V\angle\theta$$

{$\angle\theta$ - angle θ}

- **Addition & Subtraction**

(most suitable for rectangular form)

$$v_1 = x_1 + j y_1$$
$$v_2 = x_2 + j y_2$$

$$v_1 + v_2 = (x_1 + x_2) + j(y_1 + y_2)$$

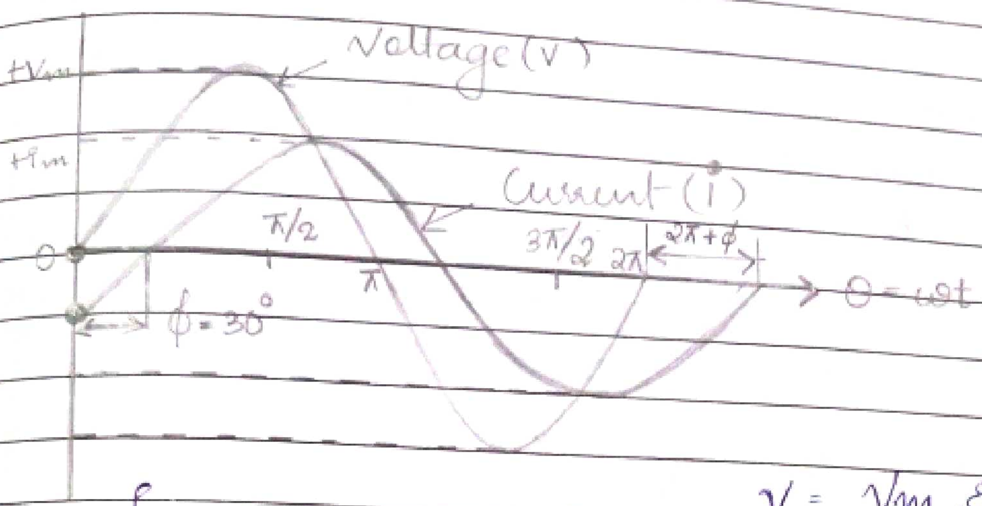
$$v_1 - v_2 = (x_1 - x_2) + j(y_1 - y_2)$$

- **Multiplication and division (polar form)**

$$v_1 = V_1\angle\theta_1$$
$$v_2 = V_2\angle\theta_2$$

$$v_1 \cdot v_2 = V_1 V_2 \angle\theta_1 + \theta_2$$

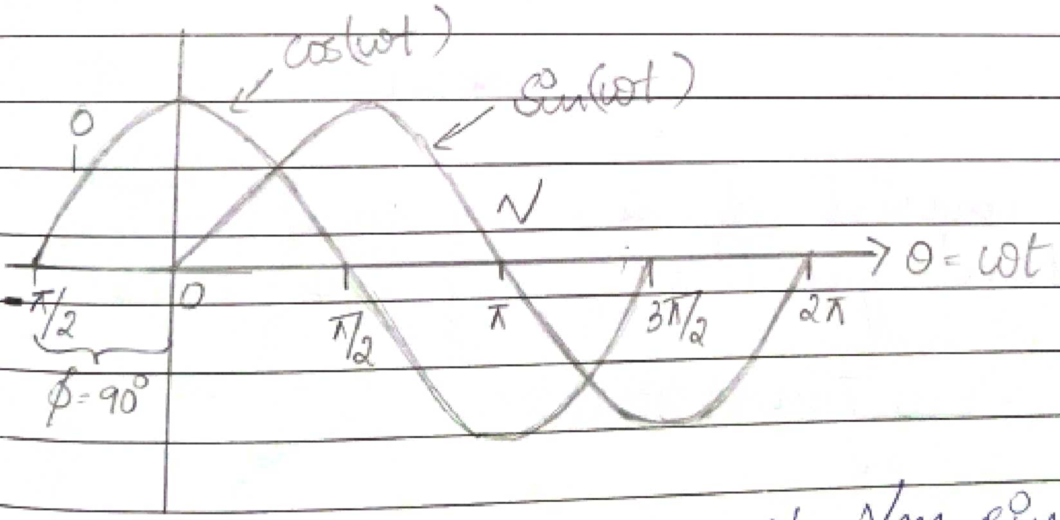
$$\frac{v_1}{v_2} = \frac{V_1}{V_2} \angle\theta_1 - \theta_2$$



{out of phase}

$v = V_m \sin(\omega t)$
reference wave

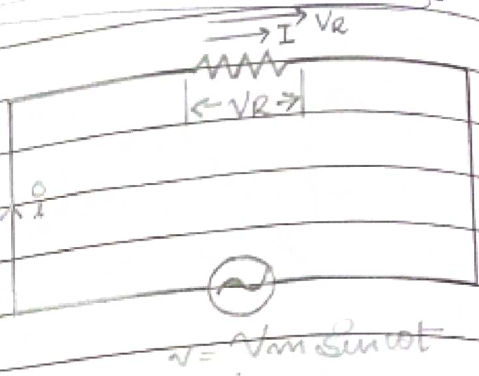
$i = I_m(\omega t - \phi)$
lagging by ϕ degree



$v = V_m \sin(\omega t)$
reference wave

$i = I_m \sin(\omega t + \phi)$
leading wave
leading by ϕ degree

AC circuit with resistance only



$v = V_m \sin \theta$
$= V_m \sin \omega t$

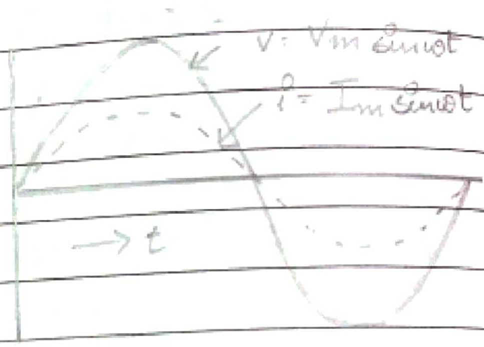
$v = iR$

$V_m \sin(\omega t) = iR$

$i = \dfrac{V_m \sin(\omega t)}{R}$

$\therefore \quad I_m = \dfrac{V_m}{R}$

$(i = I_m \sin(\omega t))$



Instantaneous power,

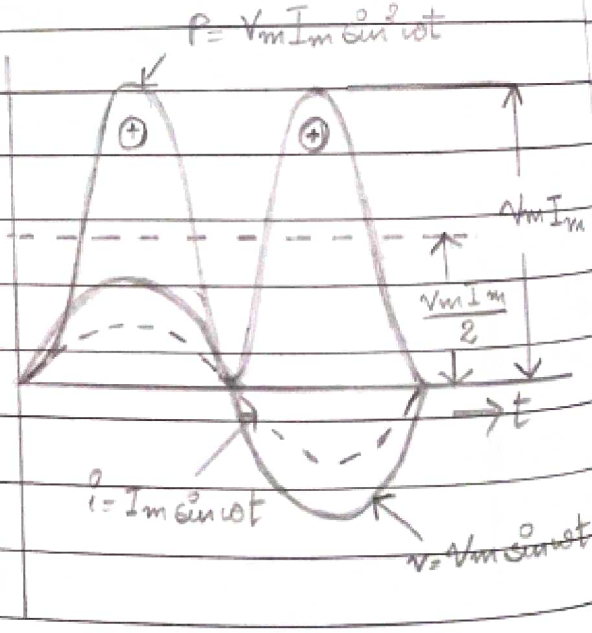
$p = vi$
$= V_m I_m \sin^2 \omega t$
$= \dfrac{V_m I_m}{2}(1 - \cos 2\omega t)$
$= \dfrac{V_m I_m}{2} - \dfrac{V_m I_m}{2}\cos(2\omega t)$

Power consist of a constant part:

$\Rightarrow \dfrac{V_m I_m}{2}$

and a fluctuating part:

$\Rightarrow \dfrac{V_m I_m}{2}\cos(2\omega t)$



frequency is double that of voltage

and current waves.

Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

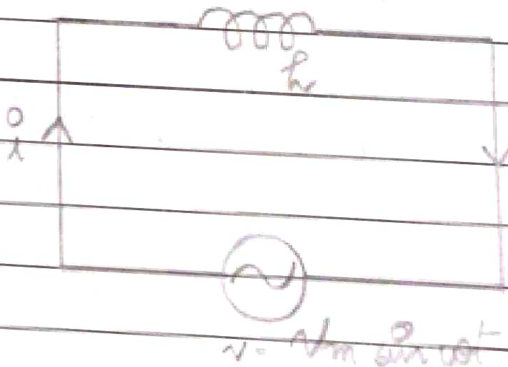
or $P = V \times I$ (Watt)

where,

V = rms value of applied voltage.
I = rms value of the current.

AC through Pure Inductive Circuit

$$v = L \frac{di}{dt}$$



Now, $v = V_m \sin(\omega t)$

$$\therefore V_m \sin(\omega t) = L \frac{di}{dt}$$

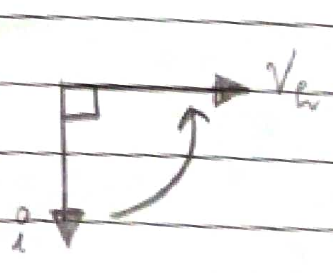
$$\therefore di = \frac{V_m}{L} \sin(\omega t) dt$$

Integrating both sides we get,

$$i = \frac{V_m}{L} \int \sin(\omega t) dt$$

$$= \frac{V_m}{L} \frac{\cos \omega t}{\omega}$$

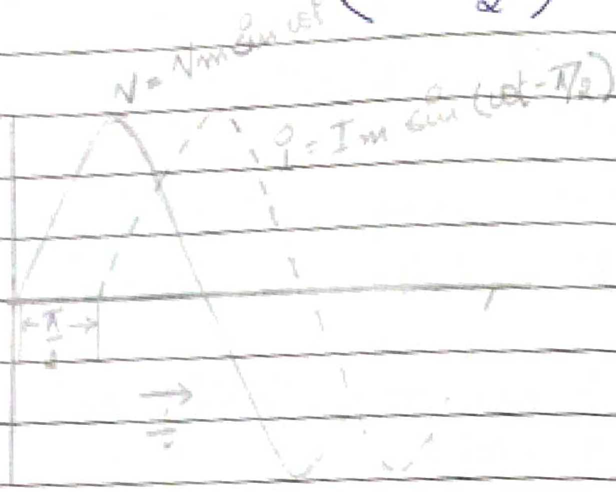
$$\therefore i = \frac{V_m}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right)$$



$$= \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Max value of i is $I_m = \frac{V_m}{\omega L}$ (when $\sin(\omega t - \frac{\pi}{2})$ is unity)

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$



Power

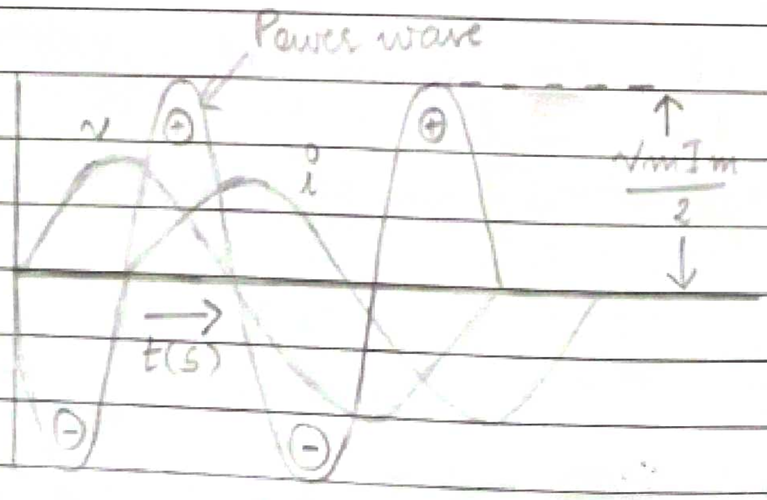
Instantaneous power = $vi = V_m I_m \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right)$

$= V_m I_m \sin(\omega t)\cos(\omega t)$

$= \frac{V_m I_m}{2} \sin(2\omega t)$

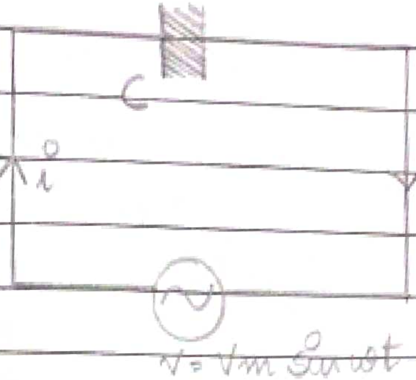
Power for whole cycle is $P = \frac{V_m I_m}{2}\int_0^{2\pi} \sin 2\omega t \, dt$

$= \underline{\underline{0}}$



AC through Pure Capacitive Circuit

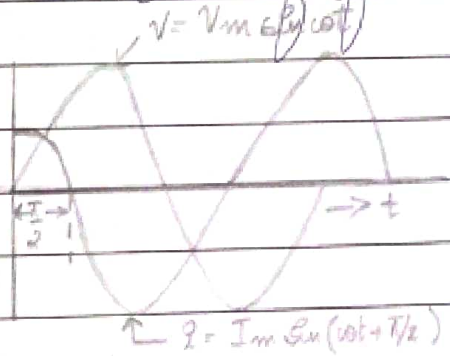
v = p.d developed between plates at any instant.
q = charge on plates at that instant



$q = Cv$ {where C is the capacitance}

$= C V_m \sin(\omega t)$ {sub $v = V_m \sin(\omega t)$}

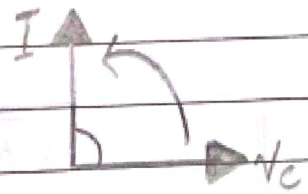
current i is given by the rate of flow of charge.



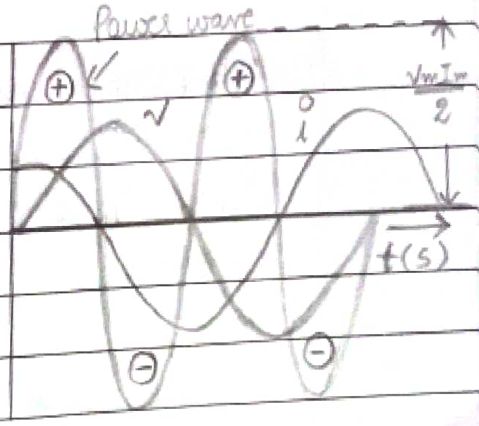
$\therefore i = \frac{dq}{dt}$

$= \frac{d(C V_m \sin \omega t)}{dt}$

$= C V_m \omega \cos \omega t$ or



$i = \frac{V_m}{1/C\omega} \sin\left(\omega t + \frac{\pi}{2}\right)$



$I_m = \frac{V_m}{1/C\omega} = \frac{V_m}{X_C}$

$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

$P = vi$
$= V_m \sin(\omega t) . I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

$= V_m I_m \sin\omega t \cos\omega t$

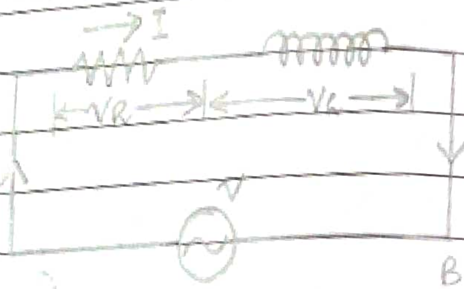
$= \frac{V_m I_m}{2} \sin 2\omega t$

Power for the whole cycle

$= \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt$

$= \underline{\underline{0}}$

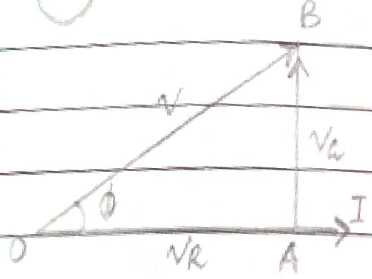
AC through RL circuit



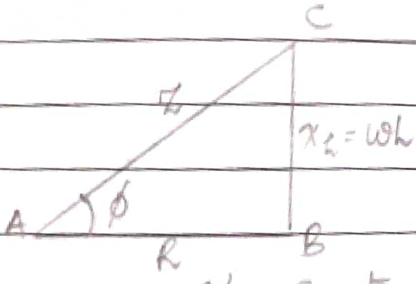
$V = \sqrt{V_R^2 + V_L^2}$

$= \sqrt{(IR)^2 + (I \cdot X_L)^2}$

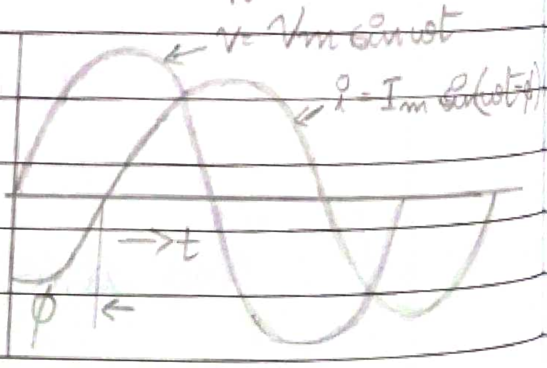
$= I\sqrt{R^2 + X_L^2}$



$I = \frac{V}{\sqrt{R^2 + X_L^2}}$



The quantity $\sqrt{R^2 + (X_L)^2}$ is known as the impedance (Z) of the circuit.



$Z^2 = R^2 + X_L^2$

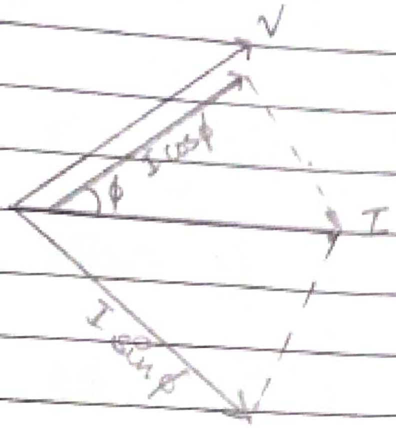
the applied voltage V leads current I by an angle ϕ such that

$$\tan\phi = \frac{V_L}{V_R} = \frac{I.X_L}{I.R}$$

$$= \frac{X_L}{R}$$

$$= \frac{\omega L}{R}$$

$$= \frac{\text{Reactance}}{\text{Resistance}}$$



$$\phi = \tan^{-1}\frac{X_L}{R}$$

I has been resolved into its two mutually perpendicular components, $I\cos\phi$ along the applied voltage V and $I\sin\phi$ in quadrature.

The mean power consumed by the circuit is given by the product of V and that component of the current I which is in phase with V.

$$P = V \times I\cos\phi$$
$$= \text{rms voltage} \times \text{rms current} \times \cos\phi$$

Q-Factor of a Coil

$$Q\text{-Factor} = \frac{1}{\text{power factor}}$$

$$= \frac{1}{\cos\phi} = \frac{Z}{R}$$

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

Power Factor

cosine of the angle of lead or lag

the ratio $\frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$

the ratio $\frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volts-amperes}} = \frac{W}{VA}$

Active and Reactive Components of Circuit Current I

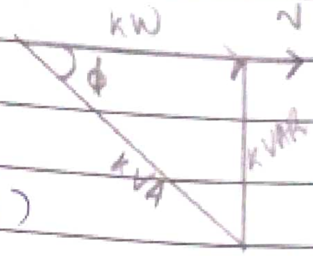
- Active component is that which is in phase with the applied voltage V i.e $I\cos\phi$.
- It is also known as 'wattful' component.
- Reactive component is that which is quadrature with V i.e $I\sin\phi$
- It is also known as 'wattless' or 'idle' component.

Active, Reactive and apparent Power

- Apparent power (S)

It is given by the product of r.m.s values of applied voltage and circuit current.

$$S = VI$$
$$= (IZ).I$$
$$S = (IZ).I = I^2 Z \text{ volt-amperes (VA)}$$



Active power (P or W)

Active component which is obtained by multiplying kVA by $\cos\phi$ and this gives power in kW.

$$P = I^2 R$$
$$= VI\cos\phi \text{ watts.}$$

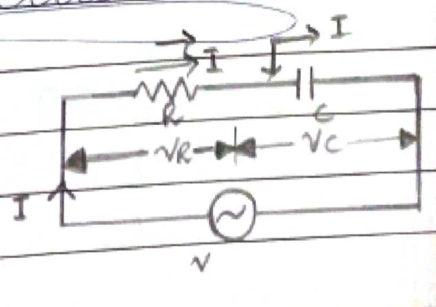
Reactive power (Q)

The reactive component known as reactive kVA and is obtained by multiplying kVA by $\sin\phi$.

It is written as kVAR (kilo VAR)

$$Q = I^2 X_L$$
$$= I^2 \times Z\sin\phi$$
$$= I \times (IZ)\sin\phi$$
$$= VI\sin\phi \text{ volt-amperes - reactive (VAR)}$$

A.C Through RC Series Circuit



$$V = \sqrt{V_R^2 + (-V_C)^2}$$
$$= \sqrt{(IR)^2 + (-IX_C)^2}$$
$$= I\sqrt{R^2 + X_C^2}$$

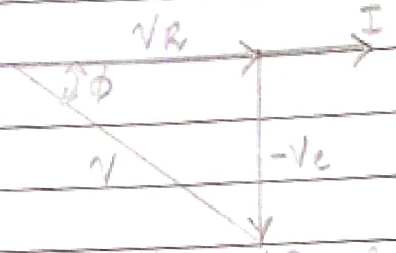
$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

$$= \frac{V}{Z}$$

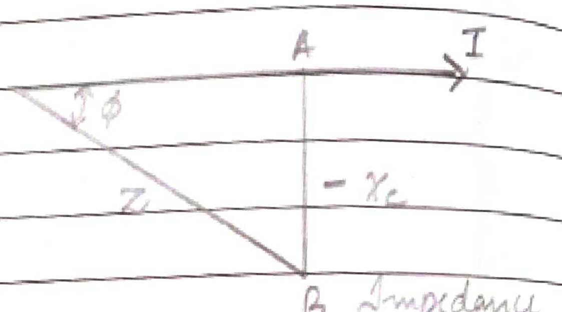
$$Z = \sqrt{R^2 + X_C^2}$$

Current leads voltage by ϕ

$$\tan\phi = \frac{-X_C}{R}$$



Voltage Triangle



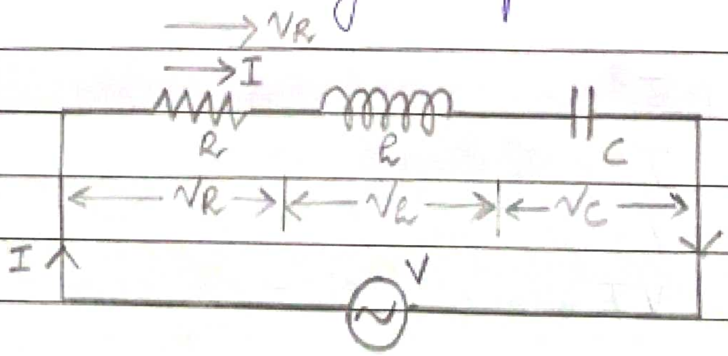
Impedance Triangle

RLC Series Circuit

$V_R = IR$ = voltage drop across R – in phase with I

$V_L = I.X_L$ = voltage drop across L – leading I by $\pi/2$

$V_C = I.X_C$ = voltage drop across C – lagging I by $\frac{\pi}{2}$.



$$OD = \sqrt{OA^2 + AD^2}$$

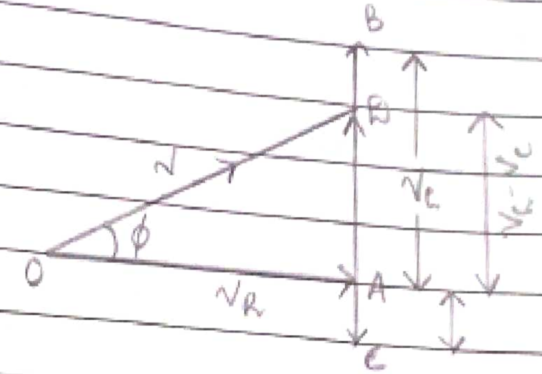
OR

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$= \frac{V}{\sqrt{R^2+X^2}} = \frac{V}{Z}$$



$$Z = \sqrt{R^2+(X_L-X_C)^2}$$

$(\text{impedance})^2 = (\text{resistance})^2 - (\text{net reactance})^2$

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$Z^2 = R^2 + X^2$$

phase angle ϕ

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$= \frac{X}{R}$$

$$= \frac{\text{net reactance}}{\text{resistance}}$$

Power factor is $\cos\phi = \frac{R}{Z}$

$$= \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$= \frac{R}{\sqrt{R^2+X^2}}$$

- resulting current in an R-L-C circuit is given by $i = I_m \sin(\omega t \pm \phi)$
- The +ve sign is to be used when current

leads i.e. $X_C > X_L$

- The -ve sign is to be used when current lags i.e when $X_L > X_C$.
- Symbolic notation : $Z = R + j(X_L - X_C)$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At phase angle is $\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

$$Z = Z \angle \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

$$= Z \angle \tan^{-1}[X/R]$$

if $V = V\angle 0$

then $I = \frac{V}{Z}$

Summary of Results of Series AC Circuits

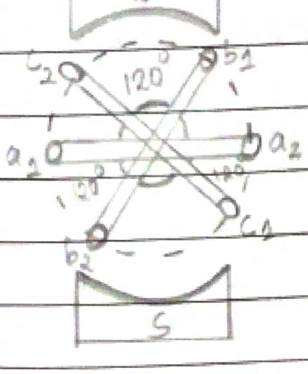
Type of impedance	Value of impedance	phase angle for current	Power factor
• Resistance only	R	0°	1
• Inductance only	ωL	90° lag	0
• Capacitance only	$1/\omega C$	90° lead	0
• Resistance and inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90^\circ$ lag	$1 > p.f > 0$ lag
• Resistance & capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90^\circ$ lead	$1 > p.f > 0$ lead
• R.L.C	$\sqrt{R^2 + (\omega L \sim 1/\omega C)^2}$	b/w 0° & 90° lag or lead	b/w 0 and unity lag or lead.

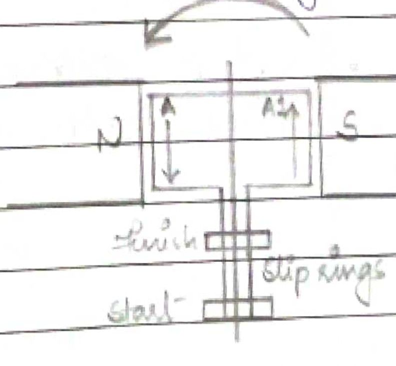
Three Phase System

Comparison between polyphase system and single phase system.

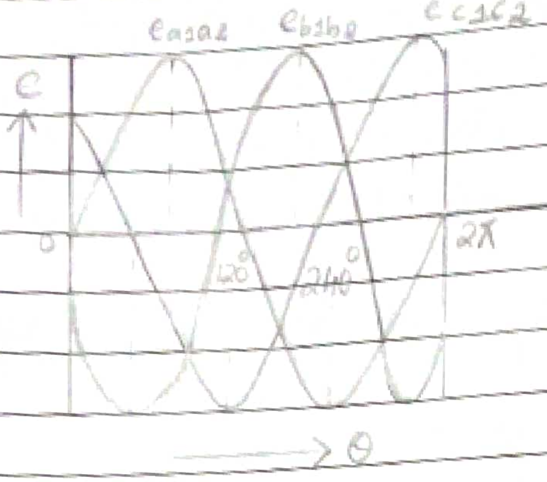
Poly phase system	Single phase system
• Power generation is cheaper	• Power generation is costly.
• High pf & efficiency	• Low pf & efficiency
• Uses less material for a given capacity.	• Uses more material
• Apparatus are economical.	• Apparatus are costly.
• Polyphase motors have uniform torque.	• Pulsating torque
• Parallel operation is very smooth.	• Not smooth.

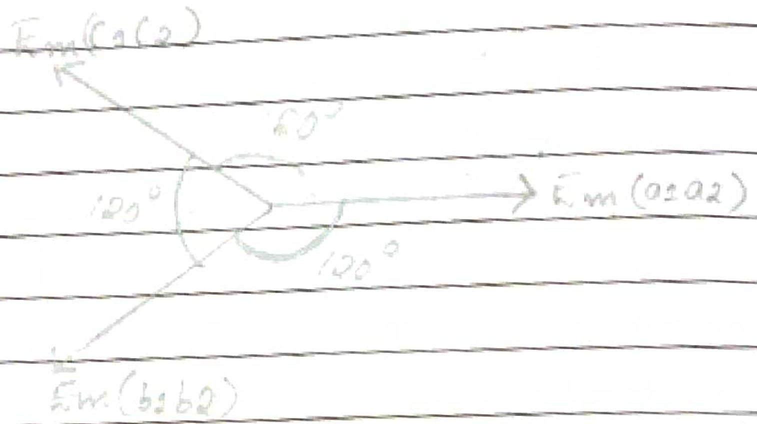
Production of Three Phase voltage





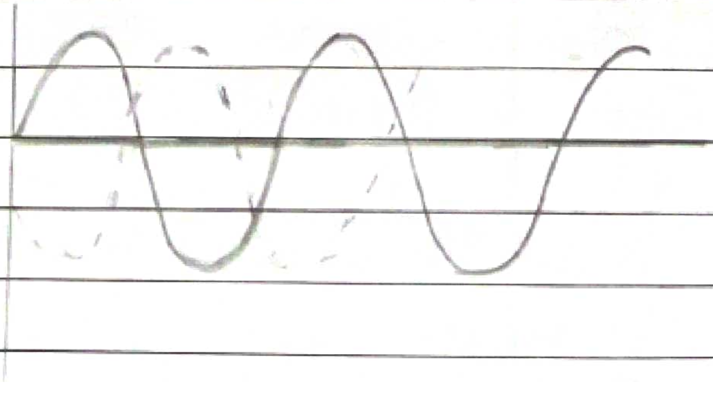
- $E_{aa1} = E_m \sin\theta$
- $E_{bb1} = E_m \sin(\theta - 120)$
- $E_{cc1} = E_m \sin(\theta - 240)$





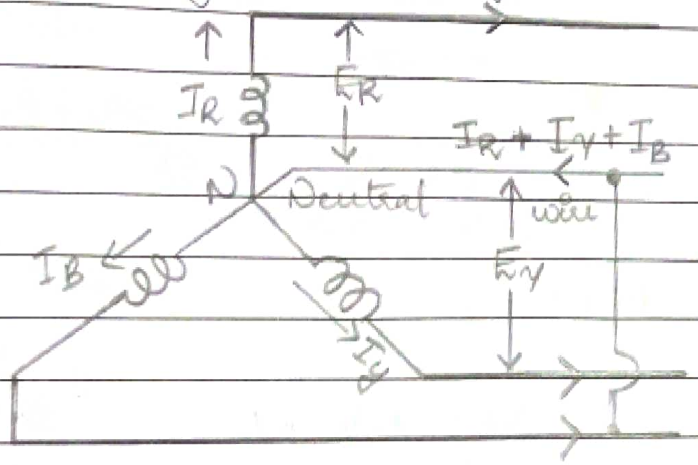
Phase Sequence or phase order

- phase sequence is meant the order in which the three phases attain their peak or maximum positive values.
- The phase sequence can be reversed by interchanging any pair of lines.

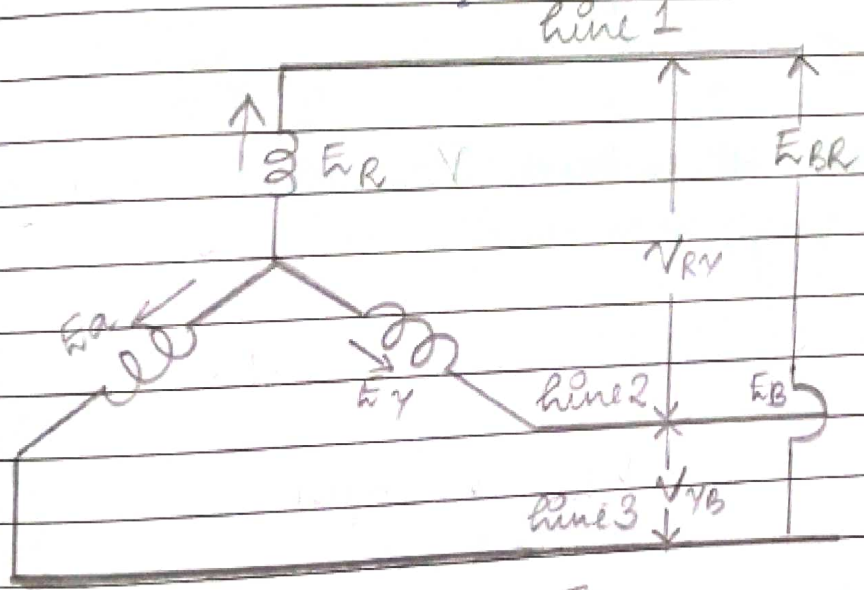


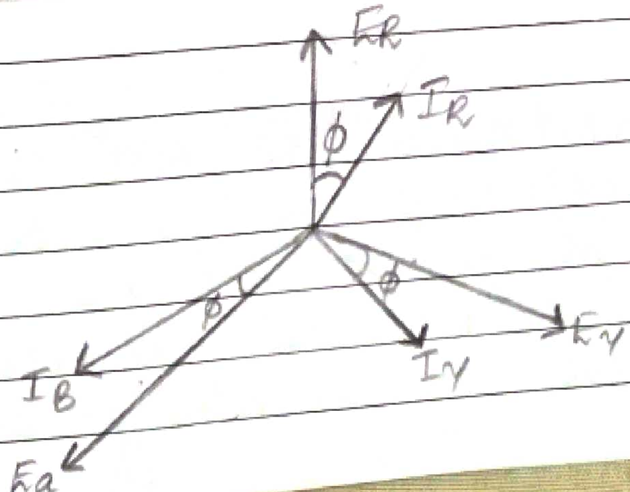
Star Connected System

The similar ends 'start' ends of three coils (it could be 'finishing' ends also) are joined together at point N.



Phase Voltage and Line voltage





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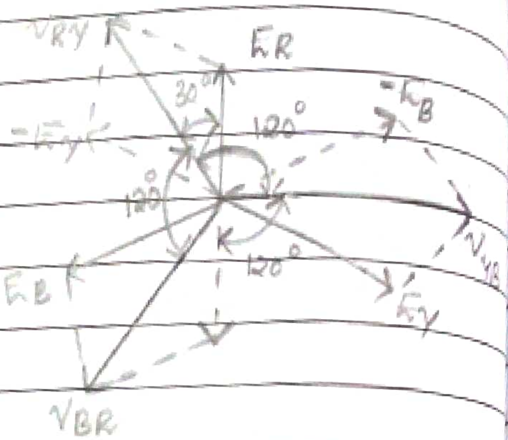
- if $E_R = E_Y = E_B$
let E_{ph} be the phase e.m.f.,

$$V_{RY} = 2 \times E_{ph} \times \cos(60°/2)$$

$$= 2 \times E_{ph} \times \cos 30°$$

$$= 2 \times E_{ph} \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}\, E_{ph}$$



Similarly,

$$V_{YB} = E_Y - E_B$$
$$= \sqrt{3} . E_{ph} \quad \{\text{vector difference}\}$$

&

$$V_{BR} = E_B - E_R$$
$$= \sqrt{3} . E_{ph}$$

where V_{RY}, V_{YB}, V_{BR} are line voltages, generally represented as V_L

Hence star connection,

$$V_L = \sqrt{3}\, E_{ph}.$$

Note:

- Line voltages are 120° apart.
- Line voltages are 30° ahead of their respective phase voltages.
- The angle between the line currents and the corresponding line voltages is

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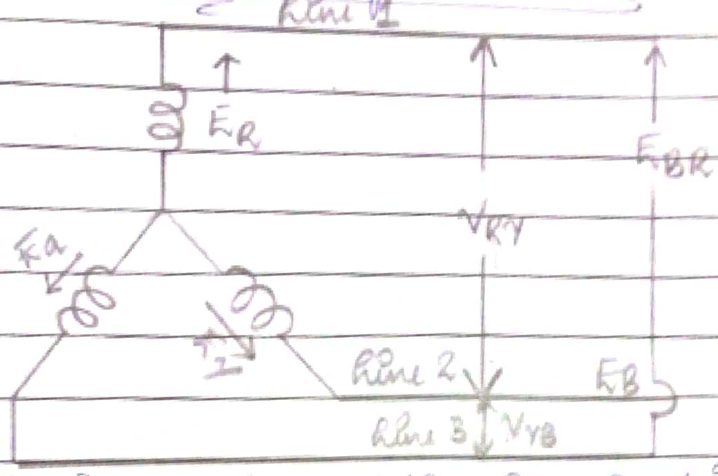
$(30+\phi)$ with current lagging.

Line Currents and phase Currents



each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 = I_R
Current in line 2 = I_Y
Current in line 3 = I_B
Since $I_R = I_Y = I_B$

Assume, I_{ph} be the phase current

$\therefore$ line current $I_L = I_{ph}$

Power

The total active or true power in the circuit is the sum of the ~~three~~ phase powers. Hence, total active power = 3 × phase power